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WAVEMAKING BY A THIN SHIP: SECOND ORDER NONLINEAR EFFECTS

By

Gedeon Dagan

October 1973

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Prepared for

Office of Naval Research
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The present work investigates second order terms of the expansion in the beam/length ratio of the potential of flow past a thin ship. The ship is in a steady translational motion, in water of infinite depth and infinite lateral extent, within its central plane of symmetry. Heave and trim are not taken into account.

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The second order spectral functions are computed in detail for the particular case of a ship with parabolical waterline and rectangular frames. Preliminary numerical results for the second order sine spectral functions for the draft/length ratio equal to 0.15 and for the Froude numbers $F_n = 0.316$ and $F_n = 0.200$ are presented and discussed.

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NOTATION

Dotted variables have dimensions; undotted variables are dimensionless (reference length is U'^2/g). Symbol ~ stands for Fourier transform. Superscripts s, b, ℓ stand for free-surface, body and waterline effects, respectively.

b'	half beam
B'	beam
$C_1, C_2^s, C_2^b, C_2^\ell, C_2$	first and second order cosine spectrum functions
$f(x,z)$	hull shape function
F_n	length Froude number
g	acceleration of gravity
G	Green function
G_r, G^w	parts of the Green function
$h(x)$	waterline shape function
h^s	auxiliary function
$I_1, I_2^s, I_2^b, I_2^\ell$	auxiliary functions
ℓ'	half length of the ship ($\ell = \ell' g/U'^2$)
L'	length of the ship
$L(\alpha, \nu)$	auxiliary function
$m(z)$	cross-section shape function

$M^{s}, M_{j}^{b}, M^{\ell}$	auxiliary functions
$p_2^{s}, p_2^{b}, p_2^{\ell}$	auxiliary functions equivalent to pressures
$q^{s}, q_{j}^{b}, q^{\ell}, q_{p}^{s}, q_{j,p}^{b}, q_{p}^{\ell}$	auxiliary functions
R	wave drag
S	submersed area in the center plane
$S_1, S_2^{s}, S_2^{b}, S_2^{\ell}, S_2$	first and second order sine spectrum functions
t'	draft
U'	ship velocity
x', y', z'	cartesian coordinates attached to the ship
α, β	longitudinal and transversal wave numbers
α_p	root of $\alpha^2 - \gamma = 0$
$\delta(z)$	Dirac function
$\bar{\epsilon} = b'/\ell'$	slenderness parameter
$\epsilon = b'g/U'^2$	slenderness parameter
$\varphi_1, \varphi_2^{s}, \varphi_2^{b}, \varphi_2^{\ell}, \varphi_2$	first and second order potentials
$\varphi_1^{w}, \varphi_2^{sw}, \varphi_2^{bw}, \varphi_2^{\ell w}, \varphi^{w}$	free wave potentials

$\gamma = (\alpha^2 + \beta^2)^{\frac{1}{2}}$, $\Gamma = [(\alpha-\nu)^2 + (\beta-\tau)^2]^{\frac{1}{2}}$	auxiliary functions
μ	coefficient of artificial viscosity
ν	integration variable
ν_p	root of $\nu^2 - \rho = 0$
$\rho = (\nu^2 + \tau^2)^{\frac{1}{2}}$	auxiliary function
τ	integration variable
$\zeta_1, \zeta_2^s, \zeta_2^b, \zeta_2^l, \zeta_2$	first and second order free surface elevation
$\zeta_1^w, \zeta_2^{sw}, \zeta_2^{bw}, \zeta_2^{lw}, \zeta_2^w$	first and second order free waves profile

ABSTRACT

The present work investigates second order terms of the expansion in the beam/length ratio of the potential of flow past a thin ship. The ship is in a steady translational motion, in water of infinite depth and infinite lateral extent, within its central plane of symmetry. Heave and trim are not taken into account.

The expressions suggested by Maruo (1966) for the second order potentials are taken as the starting point. The second order potential is split into the free-surface, body and waterline integral corrections. The associated sine and cosine spectrum functions are derived systematically by using double Fourier transforms. Computation of the value of a second order spectral function for a given transversal wave number is shown to require six integrations in general and two integrations in the case of a simple analytical hull shape function. Simplifications of spectral functions in the case of separable hull functions and symmetrical ships are discussed.

The second order spectral functions are computed in detail for the particular case of a ship with parabolical waterline and rectangular frames. Preliminary numerical results for the second order sine spectral functions for the draft/length ratio equal to 0.15 and for the Froude numbers $F_n = 0.316$ and $F_n = 0.200$ are presented and discussed.

I. INTRODUCTION

We consider the generation of waves and the associated wave resistance of a thin ship moving at constant speed in still water of infinite depth and infinite lateral extent. The usual linearized theory (Michell's theory) is known to be a first order solution in the beam/length ratio (Peters and Stoker, 1967). The general expressions of the second order potential, and the associated wave resistance, have been derived in a complete form by Wehausen (1963) and Maruo (1966). The aim of the present work is to derive a method of computing the second order effects, which has become possible with the availability of fast electronic computers.

Such computations are extremely valuable for the further advancement of the theory of wave resistance and its application. They may result in a more accurate evaluation of wave resistance of ships in those cases in which the linearized theory has been found to be unsatisfactory. Complete second order computations may also help in deciding whether some of the second order effects can be neglected in order to arrive at inconsistent, but simpler, formulations. Furthermore, by comparing measured with computed wave spectra it is possible to find out whether second order effects or viscous effects are responsible for the discrepancy between the first order solution and experiments. Finally, derivation of shapes of little wave resistance can be improved by accounting for nonlinear effects.

The second order wave resistance has been computed by Eggers (1970) for a parabolical ship. The present work parallels somehow Eggers fundamental studies, but differs in method: the second order free waves potential is derived by straightforward double Fourier transform and the wave resistance coefficient is computed from the free waves spectra.

The influence of trim and heave is not taken into account here, but the method can be used in order to incorporate these effects too.

II. GENERAL FORMULAE FOR FIRST AND SECOND ORDER POTENTIALS AND FOR WAVE RESISTANCE

We consider a symmetrical ship of length $L' = 2\ell'$, beam $B' = 2b'$ and submersed area (in the center plane, beneath the unperturbed level) S', moving at constant speed U' in the positive x' direction (Fig. 1) within its plane of symmetry. The axes x' and y' lie in the horizontal plane of the unperturbed free-surface while z' is vertical and positive upwards. $z' = -t'(x')$ describes the contour of S'.

The variables are first made dimensionless with respect to U' and U'^2/g, i.e.

$$x,y,z = x'g/U'^2,\ y'g/U'^2,\ z'g/U'^2 \quad ; \quad \varphi = \varphi'g/U'^3$$

$$\zeta = \zeta'g/U'^2 \ ; \ t = t'g/U'^2 \ ; \ \ell = \ell'g/U'^2 \ ; \ \epsilon = b'g/U'^2 \qquad [2.1]$$

where $\varphi'(x',y',z')$ is the velocity potential (the velocity is its positive gradient) and $\zeta'(x',y')$ is the free surface elevation.

The equation of the hull surface is as follows

$$y = \pm \epsilon f(x,z) \qquad (|x| < \ell, \; z > -t) \qquad [2.2]$$

We consider the translational steady motion of the ship which is kept otherwise fixed with respect to the coordinate system.

Assuming that $\epsilon = o(1)$ while $\ell = O(1)$ and $t = O(1)$ the potential is expanded in a perturbation series as follows

$$\varphi(x,y,z;\epsilon,\ell,\ldots) \sim -x + \epsilon\varphi_1(x,y,z;\ell,\ldots) + \epsilon^2\varphi_2(x,y,z;\ell,\ldots) + \ldots \qquad [2.3]$$

For the sake of completeness we give here the expressions of φ_1 and φ_2 following Maruo (1966) formulation:

$$\varphi_1(x,y,z) = \frac{1}{2\pi}\iint\limits_S \frac{\partial f}{\partial \bar{x}} G(x,y,z;\; \bar{x},0,\bar{z})\; d\bar{x}d\bar{z} \qquad [2.4]$$

$$\varphi_2 = \varphi_2^s + \varphi_2^b + \varphi_2^\ell \qquad [2.5]$$

$$\varphi_2^s(x,y,z) = \frac{1}{4\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_2^s(\bar{x},\bar{y})\; G(x,y,z;\; \bar{x},\bar{y},0)\; d\bar{x}d\bar{y} \qquad [2.6]$$

$$\varphi_2^b(x,y,z) = \frac{1}{2\pi}\iint_S p_2^b(\bar{x},\bar{z})\; G(x,y,x;\; \bar{x},0,\bar{z})\; d\bar{x}d\bar{z} \qquad [2.7]$$

$$\varphi_2^{\ell}(x,y,z) = \frac{1}{2\pi}\int_{-\ell}^{\ell} p_2^{\ell}(\bar{x})\; G(x,y,z;\; \bar{x},0,0)\; d\bar{x} \qquad [2.8]$$

where φ_2^s, φ_2^b and φ_2^{ℓ} are the free-surface, the body and the water-line integral second-order corrections, respectively, and G is the Green function. The p_2 functions have the following expressions:

$$p_2^s(\bar{x},\bar{y}) = 3\frac{\partial\varphi_1}{\partial x}\frac{\partial^2\varphi_1}{\partial x^2} + 2\frac{\partial\varphi_1}{\partial y}\frac{\partial^2\varphi_1}{\partial x\partial y} + \frac{\partial\varphi_1}{\partial x}\frac{\partial^2\varphi_1}{\partial y^2}$$

$$+ 2\frac{\partial\varphi_1}{\partial z}\frac{\partial^2\varphi_1}{\partial x\partial z} - \frac{\partial\varphi_1}{\partial x}\frac{\partial^3\varphi_1}{\partial x^2\partial z} \qquad (x=\bar{x},\; y=\bar{y},\; z=0) \qquad [2.9]$$

$$p_2^b(\bar{x},\bar{z}) = -\frac{\partial}{\partial x}\left(f\frac{\partial\varphi_1}{\partial x}\right) - \frac{\partial}{\partial z}\left(f\frac{\partial\varphi_1}{\partial z}\right) \qquad (x=\bar{x},\; y=0,\; z=\bar{z}) \qquad [2.10]$$

$$p_2^{\ell}(\bar{x}) = \frac{\partial f}{\partial x}\frac{\partial\varphi_1}{\partial z} \qquad (x=\bar{x},\; y=0,\; z=0) \qquad [2.11]$$

The Green function is given by

$$G(x,y,z;\; \bar{x},\bar{y},\bar{z}) = \frac{1}{[(x-\bar{x})^2 + (y-\bar{y})^2 + (z-\bar{z})^2]^{\frac{1}{2}}}$$

$$- \frac{1}{[(x-\bar{x})^2 + (y-\bar{y})^2 + (z+\bar{z})^2]^{\frac{1}{2}}} + G_r \qquad [2.12]$$

where G_r is regular in the lower half plane $z < 0$ and G satisfies the linearized free-surface and radiation conditions.

We shall use in the sequel the following Fourier transforms

$$\tilde{\Phi} = \frac{1}{2\pi} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\ \Phi(x,y,z)\ e^{i(\alpha x+\beta y)} \qquad [2.13]$$

$$\Phi = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\alpha \int_{-\infty}^{\infty} d\beta\ \tilde{\Phi}(\alpha,\beta,z)\ e^{-i(\alpha x+\beta y)} \qquad [2.14]$$

where α and β are the longitudinal and transversal wave numbers, respectively.

The Fourier transform (FT) of G has the following expression (see, for instance, Timman and Vossers, 1955)

$$\tilde{G} = \left\{ \frac{1}{\gamma} \left[e^{-\gamma|z-\bar{z}|} - e^{\gamma(z+\bar{z})} \right] - \frac{2\ e^{\gamma(z+\bar{z})}}{\alpha^2 - \gamma - i\mu\alpha} \right\} e^{i(\alpha\bar{x}+\beta\bar{y})} \quad [2.15]$$

where $\gamma = (\alpha^2 + \beta^2)^{\frac{1}{2}}$ is real and positive for α, β reals and μ is a positive coefficient of artificial viscosity which is let to tend to zero (μ is introduced for an easy account of integration paths in the complex plane). The three terms of $\tilde{G}$ [2.15] are the FT of the corresponding terms of G [2.12].

The FT of the different potentials are obtained by replacing G by $\widetilde{G}$ in [2.4] - [2.8]. We are interested here mainly in the potentials of the free waves for $x \to -\infty$. Only part of $\widetilde{G}$, namely

$$\widetilde{G}^{w} = -\frac{2\, e^{\gamma(z+\bar{z})}}{\alpha^2 - \gamma - i\mu\alpha}\, e^{i(\alpha\bar{x} + \beta\bar{y})} \qquad [2.16]$$

contributes to these potentials. We can write, therefore,

$$\widetilde{\varphi}_1^{w} = \frac{1}{2\pi} \iint_S \frac{\partial f}{\partial \bar{x}}\, \widetilde{G}^{w}(\bar{x},0,\bar{z};\ z)\, d\bar{x}d\bar{z} \qquad (x \to -\infty) \qquad [2.17]$$

and similar expressions for $\widetilde{\varphi}_2^{sw}$, $\widetilde{\varphi}_2^{bw}$ and $\widetilde{\varphi}_2^{\ell w}$.

To simplify notations we define the following functions

$$I_1(\alpha,\beta) = \iint_S \frac{\partial f}{\partial \bar{x}}\, e^{\gamma\bar{z}}\, e^{i\alpha\bar{x}}\, d\bar{x}d\bar{z} \qquad [2.18]$$

$$I_2^{s}(\alpha,\beta) = \frac{1}{2} \int_{-\infty}^{\infty} d\bar{x} \int_{-\infty}^{\infty} d\bar{y}\ p_2^{s}(\bar{x},\bar{y})\ e^{i(\alpha\bar{x}+\beta\bar{y})} \qquad [2.19]$$

$$I_2^{b}(\alpha,\beta) = \iint_S p_2^{b}(\bar{x},\bar{z})\ e^{\gamma\bar{z}}\ e^{i\alpha\bar{x}}\ d\bar{x}d\bar{z} \qquad [2.20]$$

$$I_2^{\ell}(\alpha,\beta) = \int_{-\ell}^{\ell} p_2^{\ell}(\bar{x})\ e^{i\alpha\bar{x}}\ d\bar{x} \qquad [2.21]$$

The free wave potential φ_1^w can be obtained from $\tilde{\varphi}_1^w$ [2.17] by inversion [2.14]. The free waves potential results from the contribution of the residues at the poles

$$\alpha = \pm \alpha_p \quad ; \quad \alpha_p = \left\{ \frac{1}{2} \left[1 + (1+4\beta^2)^{\frac{1}{2}} \right] \right\}^{\frac{1}{2}} \qquad [2.22]$$

of the quotient $1/(\alpha^2-\gamma)$ of $\tilde{G}^w$ [2.16] for integration over α. The presence of μ in [2.16] requires circumveting these poles from below. The residue of the quotient is $2\pi i\alpha_p/(2\alpha_p^2-1)$ while $\gamma(\alpha_p,\beta) = \gamma(-\alpha_p,\beta) = \alpha_p^2$.

Hence, the free waves potential φ_1^w may be expressed with the aid of [2.16], [2.17] and [2.18] as follows

$$\varphi_1^w(x,y,z) = \frac{1}{\pi}\int_{-\infty}^{\infty} \left\{ -i\left[I_1(\alpha_p,\beta) - I_1(-\alpha_p,\beta)\right] \cos \alpha_p x - \right.$$

$$\left. - \left[I_1(\alpha_p,\beta) + I_1(-\alpha_p,\beta)\right] \sin \alpha_p x \right\} \frac{\alpha_p}{2\alpha_p^2-1} e^{\alpha_p^2 z} \cos \beta y d\beta \qquad [2.23]$$

the expressions of φ_2^{sw}, φ_2^{bw} and $\varphi_2^{\ell w}$ being similar to [2.23] with I_1 replaced by I_2^s, I_2^b and I_2^{ℓ} [2.19] - [2.21], respectively. Cos βy has been substituted for $e^{i\beta y}$ in [2.23] because of the symmetry of the flow with respect to $y = 0$. Equations [2.18] - [2.21] show indeed that $I_j(\alpha,\beta) - I_j(-\alpha,\beta)$ is purely imaginary while $I_j(\alpha,\beta) + I_j(-\alpha,\beta)$ is real ($j = 1,2$), so that φ_j^w are real.

The equation of the free-surface has an expansion similar to [2.3]

$$\zeta(x,y;\epsilon,\ell,\ldots) \sim \epsilon\zeta_1(x,y;\ell,\ldots) + \epsilon^2\zeta_2(x,y;\ell,\ldots) + \ldots \tag{2.24}$$

The equation of the free waves profile is obtained from the potential as follows

$$\zeta_1^W(x,y) = \frac{\partial\varphi_1^W(x,y,0)}{\partial x} \tag{2.25}$$

$$\zeta_2^W(x,y) = \zeta_2^{sw} + \zeta_2^{bw} + \zeta_2^{\ell w} = \frac{\partial\varphi_2^{sw}(x,y,0)}{\partial x} + \frac{\partial\varphi_2^{bw}(x,y,0)}{\partial x} + \frac{\partial\varphi_2^{\ell w}(x,y,0)}{\partial x} \tag{2.26}$$

It is worthwhile to mention here that ζ_1^W differs from ζ_1 by local terms which decay algebraically like $(x^2+y^2)^{-2}$ for $x \to -\infty$, while the local terms in ζ_2 are of order $(x^2+y^2)^{-\frac{1}{2}}$.

It is seen from [2.25] and [2.23] that one can write for the free waves profile

$$\zeta_1^W(x,y) = \frac{1}{\pi}\int_{-\infty}^{\infty} [S_1(\beta) \sin \alpha_p x + C_1(\beta) \cos \alpha_p x] \frac{\alpha_p^2}{2\alpha_p^2 - 1} \cos \beta y d\beta \tag{2.27}$$

where the sine and cosine spectrum functions $S_1(\beta)$ and $C_1(\beta)$ are given by

$$S_1(\beta) - iC_1(\beta) = 2iI_1(\alpha_p,\beta) \qquad [2.28]$$

Similar expressions are obtained for ζ_2^{sw}, ζ_2^{bw} and $\zeta_2^{\ell w}$ with S_1, C_1 replaced by S_2^s, C_2^s, S_2^b, C_2^b, S_2^ℓ, C_2^ℓ, respectively, in [2.27] and I_1 by I_2^s, I_2^b and I_2^ℓ in [2.28].

Finally, the dimensionless wave drag $R = R'g^2/\rho U'^4$ can be written, on the basis of the first and second order potentials, as follows

$$R \sim \frac{1}{2\pi}\int_{-\infty}^{\infty} [(\epsilon S_1 + \epsilon^2 S_2)^2 + (\epsilon C_1 + \epsilon^2 C_2)^2] \frac{\alpha_p^2}{2\alpha_p^2-1}\, d\beta \qquad [2.29]$$

Expending R in an ϵ series yields

$$R \sim \epsilon^2 R_1 + \epsilon^3 R_2 + O(\epsilon^4) \qquad [2.30]$$

where

$$R_1 = \frac{1}{2\pi}\int_{-\infty}^{\infty} (S_1^2 + C_1^2) \frac{\alpha_p^2}{2\alpha_p^2-1}\, d\beta \qquad [2.31]$$

$$R_2 = \frac{1}{\pi}\int_{-\infty}^{\infty} (S_1 S_2 + C_1 C_2) \frac{\alpha_p^2}{2\alpha_p^2-1}\, d\beta \qquad [2.32]$$

R [2.29] is always positive, whether R [2.30] may become negative if the second order effects are exceedingly large. Using [2.29] is, however, inconsistent because it includes contributions $O(\epsilon^4)$.

All the formulae of this section are well documented in the literature (for instance in Wehausen, 1973) and have been given here for the sake of completeness and clarity of notation.

The purpose of the present work is to derive the second order spectrum functions $S_2(\beta)$ and $C_2(\beta)$ for ships of given shape.

III. THE COMPUTATION OF THE SECOND ORDER FREE WAVES SPECTRUM FUNCTIONS

Equations [2.19] - [2.21] render the functions needed for the computation of the spectral functions in terms of the first order solution φ_1. Considerable simplifications in number of integrations and smoothness of integrands may be achieved however, by further elaboration of the integrals of [2.19] - [2.21].

1. <u>The Complete Expression of $\tilde{\varphi}_1$</u>

Equations [2.16] and [2.17] render the expressions of the FT of the first order free waves potential. The second order terms, however, are based on the complete expression of φ_1, i.e., on the local as well as the far waves term. It is useful, therefore, to write $\tilde{\varphi}_1$ and its derivatives in an explicit form. From [2.4] and [2.15] we have

$$\tilde{\varphi}_1(\alpha,\beta,z) = \frac{1}{2\pi}\iint_S \frac{\partial f(\bar{x},\bar{z})}{\partial \bar{x}} \left\{ \frac{1}{\gamma}\left[e^{-\gamma|z-\bar{z}|} - e^{\gamma(z+\bar{z})}\right] - \right.$$

$$\left. - \frac{2e^{\gamma(z+\bar{z})}}{\alpha^2-\gamma-i\mu\alpha} \right\} e^{i\alpha\bar{x}}\, d\bar{x}d\bar{z} \qquad [3.1]$$

We shall consider here smooth ship shapes, the highest conceivable singularity of f being associated with a flat bottom, where f has a finite jump for z = -t. It is seen from [3.1] that $\tilde{\varphi}_1$ and $\widetilde{(\partial\varphi_1/\partial z)}$ are then continuous for z = -t and $\widetilde{(\partial^2\varphi_1/\partial z^2)}$ has a finite discontinuity at z = -t. This observation will be used in the sequel.

φ_2^s [2.6] and φ_2^ℓ [2.8] are based on the equations of $\varphi_1(x,y,0)$ and its derivatives. For z = 0 we obtain from [3.1] the following relationships

$$\tilde{\varphi}_1(\alpha,\beta,0) = \tilde{\varphi}_1^W(\alpha,\beta,0) = -\frac{4}{\pi(\alpha^2-\gamma-i\mu\alpha)} I_1(\alpha,\beta) \qquad [3.2]$$

$$\frac{\widetilde{\partial\varphi_1}}{\partial x} = -i\alpha\tilde{\varphi}_1^W;\ \frac{\widetilde{\partial\varphi_1}}{\partial y} = -i\beta\tilde{\varphi}_1^W;\ \frac{\widetilde{\partial\varphi_1}}{\partial z} = \alpha^2\tilde{\varphi}_1^W \qquad (z = 0) \qquad [3.3]$$

and similarly for higher order derivatives.

φ_2^b [2.7] is based on $\partial\varphi_1/\partial x$ and $\partial\varphi_1/\partial z$ along S (y=0). For the FT of these components we obtain from [3.1]

$$\frac{\widetilde{\partial\varphi_1}}{\partial x} = -\, i\alpha\tilde{\varphi}_1 \qquad (y = 0) \qquad [3.4]$$

$$\frac{\widetilde{\partial\varphi_1}}{\partial z} = \frac{1}{2\pi}\iint\limits_S \frac{\partial f}{\partial \bar{x}} \left\{ -\left[e^{-\gamma|z-\bar{z}|}\text{sign}\,(z-\bar{z}) + e^{\gamma(z+\bar{z})}\right] - \right.$$

$$\left. - \frac{2\gamma e^{\gamma(z+\bar{z})}}{\alpha^2-\gamma-i\mu\alpha} \right\} e^{i\alpha\bar{x}}\, d\bar{x}d\bar{z} \qquad [3.5]$$

2. <u>The Computation of S_2^s and C_2^s</u>.

We consider first the free surface second order effect. The function $I_2^s(\alpha,\beta)$ [2.19] is proportional to the FT of $p_2^s(\bar{x},\bar{y})$ [2.9]. Using the convolution theorem in its following symmetrical form

$$\widetilde{\omega(x,y)\ \psi(x,y)} = \frac{1}{4\pi}\int_{-\infty}^{\infty} d\nu \int_{-\infty}^{\infty} d\tau\ [\tilde{\omega}(\alpha-\nu,\beta-\tau)\tilde{\psi}(\nu,\tau)+\tilde{\omega}(\nu,\tau)\tilde{\psi}(\alpha-\nu,\beta-\tau)] \qquad [3.6]$$

where ω and ψ are arbitrary functions having FT and ν,τ are integration variables, we may write with the aid of [2.9] and [3.3]

$$I_s^S(\alpha,\beta) = \frac{1}{4}\int_{-\infty}^{\infty} d\nu \int_{-\infty}^{\infty} d\tau \; h^S(\alpha,\beta,\nu,\tau)\tilde{\varphi}_i^W(\nu,\tau,0)\tilde{\varphi}_i^W(\alpha-\nu,\beta-\tau,0) \qquad [3.7]$$

with

$$h^S(\alpha,\beta,\nu,\tau) = 3\alpha\nu(\alpha-\nu)+2\alpha\tau(\beta-\tau)+\nu(\beta-\tau)^2+(\alpha-\nu)^2\nu^2-2\alpha\nu^2(\alpha-\nu)^2-$$

$$-\ \nu(\alpha-\nu)\ [\nu^3+(\alpha-\nu)^3] \qquad [3.8]$$

By substituting in [3.7] the expressions of $\tilde{\varphi}_i^W$ [3.2, 2.18] we may rewrite [3.7] as follows

$$I_s^S(\alpha,\beta) = \int_{-\infty}^{\infty} d\nu \int_{-\infty}^{\infty} d\tau \; b^S(\alpha,\beta,\nu,\tau)q^S(\alpha,\beta,\nu,\tau) \qquad [3.9]$$

where

$$q^S(\alpha,\beta,\nu,\tau) = \frac{h^S(\alpha,\beta,\nu,\tau)}{(\nu^2-\rho-i\mu\nu)\ [(\alpha-\nu)^2 - \Gamma - i\mu(\alpha-\nu)]} \qquad [3.10]$$

$$b^S(\alpha,\beta,\nu,\tau) = \frac{1}{4\pi^2}\left(\iint_S \frac{\partial f}{\partial \bar{x}}\, e^{\Gamma\bar{z}}\, e^{i(\alpha-\nu)\bar{x}} d\bar{x}d\bar{z}\right)\left(\iint_S \frac{\partial f}{\partial \bar{x}}\, e^{\rho\bar{\bar{z}}}\, e^{i\nu\bar{\bar{x}}} d\bar{\bar{x}}d\bar{\bar{z}}\right) \qquad [3.11]$$

$$\rho = (\nu^2+\tau^2)^{\frac{1}{2}} \qquad \Gamma = [(\alpha-\nu)^2 + (\beta-\tau)^2]^{\frac{1}{2}} \qquad [3.12]$$

and ρ, Γ are real and positive for $\nu,\tau,\alpha-\nu,\beta-\tau$ real.

Due to the use of [3.6] the function $q^s(\alpha,\beta,\nu,\tau)$ and $b^s(\alpha,\beta,\nu,\tau)$ have the following symmetry property: under a change of variables $\alpha-\nu = \bar{\nu}$, $\beta-\tau = \bar{\tau}$

$$q^s(\alpha,\beta,\nu,\tau) = q^s(\alpha,\beta,\bar{\nu},\bar{\tau})$$
$$b^s(\alpha,\beta,\nu,\tau) = b^s(\alpha,\beta,\bar{\nu},\bar{\tau}) \qquad [3.13]$$

The contribution of q^s in [3.9] under integration over the variable ν, for instance, may be further split into an imaginary part resulting from the semiresidue at the poles of q^s [3.10] and a real part from a Cauchy principal value. The poles of the quotient $1/(\nu^2-\rho)$ of q_s are at $\nu = \pm \nu_p$, where similarly to [2.22]

$$\nu_p = \left\{\frac{1}{2}\left[1 + (1+4\tau^2)^{\frac{1}{2}}\right]\right\}^{\frac{1}{2}} \qquad [3.14]$$

and they have to be circumvented from below. The other two poles are associated with the quotient $1/[(\alpha-\nu)^2 - \Gamma]$ in [3.10], but due to the symmetry of q^s [3.13] it is enough to account for the semiresidue only at $\pm \nu_p$ and to take the result twice. Hence,

$$I^s(\alpha,\beta) = \pi i \int_{-\infty}^{\infty} d\tau\,[b^s(\alpha,\beta,\nu_p,\tau)q_p^s(\alpha,\beta,\nu_p,\tau)+b^s(\alpha,\beta,-\nu_p,\tau)q_p^s(\alpha,\beta,-\nu_p,\tau)]$$

$$+\int_{-\infty}^{\infty} d\tau \fint_0^{\infty} d\nu\,[b^s(\alpha,\beta,\nu,\tau)q^s(\alpha,\beta,\nu,\tau)+b^s(\alpha,\beta,-\nu,\tau)q^s(\alpha,\beta,-\nu,\tau)] \qquad [3.15]$$

with

$$q_p^s(\alpha,\beta,\nu,\tau) = \frac{2\nu h^s(\alpha,\beta,\nu,\tau)}{[(\alpha-\nu)^2-\Gamma]\,(2\nu^2-1)} \qquad [3.16]$$

The symbol $\fint$ stands for the Cauchy principal value at the poles of the integrand. Due to [3.13] we have the following symmetry properties

$$b^s(\alpha,\beta,\pm\nu,\tau) = -\bar{b}_s(-\alpha,\beta,\mp\nu,\tau)$$
$$q_p^s(\alpha,\beta,\pm\nu,\tau) = q_p^s(-\alpha,\beta,\mp\nu,\tau) \qquad [3.17]$$
$$q^s(\alpha,\beta,\pm\nu,\tau) = -q^s(-\alpha,\beta,\mp\nu,\tau)$$

where $\bar{b}_s$ is complex conjugate of b_s.

With the aid of [3.15] and [3.17] we may write the spectral functions in their final form as follows

$$S_a^s(\beta) - iC_a^s(\beta) = 2iI_a^s(\alpha_p,\beta,\nu,\tau) = -2\pi\int_{-\infty}^{\infty}\left[b^s(\alpha_p,\beta,\nu_p,\tau)\,q_p^s(\alpha_p,\beta,\nu_p,\tau)\, -\right.$$

$$\left. - \,\bar{b}^s(-\alpha_p,\beta,\nu_p,\tau)\,q_p^s(-\alpha_p,\beta,\nu_p,\tau)\right]d\tau \, +$$

$$+ \, 2i\int_{-\infty}^{\infty}d\tau\int_{0}^{\infty}d\nu\left[b^s(\alpha_p,\beta,\nu,\tau)\,q^s(\alpha_p,\beta,\nu,\tau)\, +\right.$$

$$\left. + \,\bar{b}^s(-\alpha_p,\beta,\nu,\tau)\,q^s(-\alpha_p,\beta,\nu,\tau)\right] \qquad [3.18]$$

The functions q^s [3.10, 3.12, 3.18] and q_p^s [3.16] are real and do not depend on the shape of the ship. b^s [3.11] is generally complex and depends quadratically on $\partial f/\partial x$.

3. The Computation of S_a^b and C_a^b

We consider now the second order body effect. First, we transform the expression of I_a^b [2.20, 2.10] by integrating by parts and by assuming that along the contour of S $f = 0$ (the result being valid even for a flat bottom) to obtain

$$I_a^b(\alpha,\beta) = I_a^{bc}(\alpha,\beta) + I_a^{b\ell}(\alpha,\beta) = \iint_S f(\bar{x},\bar{z})\left[i\alpha\,\frac{\partial\varphi_1(\bar{x},0,\bar{z})}{\partial\bar{x}}\, +\right.$$

$$\left. + \,\gamma\,\frac{\partial\varphi_1(\bar{x},0,\bar{z})}{\partial\bar{z}}\right]e^{\gamma\bar{z}}\,e^{i\alpha\bar{x}}\,d\bar{x}\,d\bar{z} - \int_{\ell}^{-\ell} f(\bar{x},0)\,\frac{\partial\varphi_1(\bar{x},0,0)}{\partial\bar{z}}\,e^{i\alpha\bar{x}}\,d\bar{x}$$

$$[3.19]$$

Hence I_2^b has been split into an integral over the center plane I_2^{bc} and a waterline integral $I_2^{b\ell}$.

We transform the expression of I_2^{bc} by substituting for $\partial\varphi_1/\partial\bar{x}$ and $\partial\varphi_1/\partial\bar{z}$ in [3.19]

$$\frac{\partial\varphi_1(\bar{x},0,\bar{z})}{\partial\bar{x}} = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu \int_{-\infty}^{\infty} d\tau \, \widetilde{\frac{\partial\varphi_1}{\partial\bar{x}}} \, e^{-i\nu\bar{x}} \tag{3.20}$$

$$\frac{\partial\varphi_1(\bar{x},0,\bar{z})}{\partial\bar{z}} = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu \int_{-\infty}^{\infty} d\tau \, \widetilde{\frac{\partial\varphi_1}{\partial\bar{z}}} \, e^{-i\nu\bar{x}} \tag{3.21}$$

and after using [3.4, 3.5, 3.1] and an integration by parts over $\bar{x}$ we obtain

$$I_2^{bc}(\alpha,\beta) = \int_{-\infty}^{\infty} d\nu \int_{-\infty}^{\infty} d\tau \sum_{j=I}^{j=IV} q_j^b b_j^b \tag{3.22}$$

where

$$q_I^b = -\frac{\gamma}{\alpha-\nu} \; ; \; q_{II}^b = \frac{\alpha\nu}{\rho(\alpha-\nu)} \; ; \; q_{III}^b = -\frac{\gamma\rho+\alpha\nu}{\rho(\alpha-\nu)} \; ; \; q_{IV}^b = -\frac{2(\gamma\rho+\alpha\nu)}{(\alpha-\nu)(\nu^2-\rho-i\mu\nu)} \tag{3.23}$$

$$\begin{pmatrix} b^b_I \\ b^b_{II} \\ b^b_{III} = b^b_{IV} \end{pmatrix} = \frac{i}{4\pi^2}\iint_S d\bar{x}d\bar{z}\,\frac{\partial f}{\partial \bar{x}}\,e^{\gamma\bar{z}}e^{i(\alpha-\nu)\bar{x}}\iint_S d\bar{\bar{x}}d\bar{\bar{z}}\,\frac{\partial f}{\partial \bar{\bar{x}}}\,e^{i\nu\bar{\bar{x}}}\begin{pmatrix} e^{-\rho|\bar{z}-\bar{\bar{z}}|}\operatorname{sign}(\bar{z}-\bar{\bar{z}}) \\ e^{-\rho|\bar{z}-\bar{\bar{z}}|} \\ e^{\rho|\bar{z}+\bar{\bar{z}}|} \end{pmatrix} \qquad [3.24]$$

Next we consider $I_s^{b\ell}$ [3.19] which under substitution of [3.21] and after using [3.1, 3.2] and an integration by parts over $\bar{x}$ becomes

$$I_s^{bc}(\alpha,\beta) = \int_{-\infty}^{\infty} d\nu \int_{-\infty}^{\infty} d\tau\ q^b_V\ b^b_V \qquad [3.25]$$

where

$$q^b_V = \frac{2\nu^2}{(\alpha-\nu)\ (\nu^2-\rho-i\mu\nu)} \qquad [3.26]$$

$$b^b_V = \frac{i}{4\pi^2}\left(\int_{-\ell}^{\ell} d\bar{x}\ \frac{\partial f(\bar{x},0)}{\partial \bar{x}}\ e^{i(\alpha-\nu)\bar{x}}\right)\left(\iint_S d\bar{\bar{x}}d\bar{\bar{z}}\ \frac{\partial f(\bar{\bar{x}},\bar{\bar{z}})}{\partial \bar{\bar{x}}}\ e^{\rho\bar{\bar{z}}}e^{i\nu\bar{\bar{x}}}\right) \qquad [3.27]$$

Summarizing, I_s^b [3.19] has the following expression

$$I_2^b(\alpha,\beta) = \sum_{j=I}^{V} \int_{-\infty}^{\infty} d\nu \int_{-\infty}^{\infty} d\tau \; q_j^b(\alpha,\beta,\nu,\tau) b_j^b(\alpha,\beta,\nu,\tau) \qquad [3.28]$$

Equation [3.28] has the same structure as [3.9]; we proceed, therefore, along the same lines: we separate the contributions of q_j^b at the poles $\nu = \pm\nu_p$ [3.14] and on the rest of the ν axis and use the symmetry properties of q_j^b and b_j^b which are similar to [3.17]. The final result for the spectral functions is, like in [3.18],

$$S_2^b(\beta)-iC_2^b(\beta) = 2iI_2^b(\alpha_p,\beta,\nu,\tau) =$$

$$= -4\pi \sum_{j=IV}^{j=V} \int_0^{\infty} d\tau [b_j^b(\alpha_p,\beta,\nu_p,\tau) q_{j,p}^b(\alpha_p,\beta,\nu_p,\tau) -$$

$$- \bar{b}_j^b(-\alpha_p,\beta,\nu_p,\tau) q_{j,p}^b(-\alpha_p,\beta,\nu_p,\tau)] +$$

$$+ 4i \sum_{j=I}^{V} \int_0^{\infty} d\tau \int_0^{\infty} d\nu [b_j^b(\alpha_p,\beta,\nu,\tau) q_j^b(\alpha_p,\beta,\nu,\tau) +$$

$$+ \bar{b}_j^b(-\alpha_p,\beta,\nu,\tau) q_j^b(-\alpha_p,\beta,\nu,\tau)] \qquad [3.29]$$

where

$$q_{IV,p}^b = - \frac{2\nu(\gamma\rho+\alpha\nu)}{(\alpha-\nu)\ (2\nu^2-1)} \quad ; \quad q_{V,p}^b = \frac{2\nu^3}{(\alpha-\nu)\ (2\nu^2-1)} \qquad [3.30]$$

The principal value in the last integral of [3.29] is at $\nu = \nu_p$ solely. The singularity at $\nu = \alpha$ of q_j^b [3.23, 3.26] and $q_{j,p}^b$ [3.30] is removable since $b_j^b = O(\alpha - \nu)$ for $\alpha - \nu \to 0$. Like in [3.18] the function q_j^b and $q_{j,p}^b$ in [3.29] are independent of the shape of the ship while b_j^b depend quadratically upon $\partial f/\partial x$.

4. The Computation of S_s^ℓ and C_s^ℓ

Last we consider the waterline integral second order effect. By substituting [3.20] in I_s^ℓ [2.21] and after using [3.3, 3.2, 2.18] we obtain

$$I_s^\ell(\alpha,\beta) = \int_{-\infty}^{\infty} d\nu \int_{-\infty}^{\infty} d\tau \; q^\ell(\nu,\tau) b^\ell(\alpha,\beta,\nu,\tau) \qquad [3.31]$$

where

$$q^\ell(\nu,\tau) = \frac{2\nu}{\nu^2 - \rho - i\mu\nu} \qquad [3.32]$$

$$b^\ell(\alpha,\beta,\nu,\tau) = b_\nu^b = \frac{1}{4\pi^2}\left(\int_{-\ell}^{\ell} d\bar{x}\, \frac{\partial f(\bar{x},0)}{\partial \bar{x}}\, e^{i(\alpha-\nu)\bar{x}}\right)\left(\iint_S d\bar{\bar{x}}\, d\bar{\bar{z}}\, \frac{\partial f(\bar{\bar{x}},\bar{\bar{z}})}{\partial \bar{\bar{x}}}\, e^{\rho\bar{\bar{z}}} e^{i\nu\bar{\bar{x}}}\right) \qquad [3.33]$$

By the same procedure of splitting the integral over ν in [3.31] as in the preceding sections we arrive at the final equation of the spectral functions

$$S_2^{\ell}(\beta)-iC_2^{\ell}(\beta) = 2iI_2^{\ell}(\alpha_p,\beta) =$$

$$= -4\pi\int_0^\infty q_p^{\ell}(\nu_p)[b^{\ell}(\alpha_p,\beta,\nu_p,\tau)-\overline{b}^{\ell}(-\alpha_p,\beta,\nu_p,\tau)]d\tau +$$

$$+ 4i\int_0^\infty d\tau\int_0^\infty d\nu\; q^{\ell}(\nu,\tau)[b^{\ell}(\alpha_p,\beta,\nu,\tau)+\overline{b}^{\ell}(-\alpha_p,\beta,\nu,\tau)] \quad [3.34]$$

where

$$q_p^{\ell}(\nu) = \frac{2\nu^2}{2\nu^2-1} \quad [3.35]$$

The structure of [3.34] is similar to that of [3.18] and [3.29] and it has the same properties as the other spectral functions.

5. Summary

Equations [3.18], [3.29] and [3.33] render the various second order free waves spectrum functions in a compact and similar form. The total spectrum functions are obtained by summation

$$C_2(\beta)-iS_2(\beta) = C_2^s+C_2^b+C_2^{\ell}-i(S_2^s+S_2^b+S_2^{\ell}) \quad [3.36]$$

In the most general case in which the shape function $f(x,z)$ is not given in an analytical form, two numerical integrations, over $\bar{x}$ and $\bar{z}$, are required in order to obtain a point on the curves of $C_1(\beta)$ or $S_1(\beta)$ [2.28, 2.18]. To compute the value of one of the components of C_2 or S_2 [3.35], for a given β, six numerical integrations are generally needed namely over $\bar{x}$, $\bar{z}$, $\bar{\bar{x}}$, $\bar{\bar{z}}$, ν and τ. This amounts to a huge volume of computations and current calculations of second order effect for ships of arbitrary shapes do not seem, therefore, to be of practical interest at the present time.

Considerable simplifications are achieved, however, if $f(x,z)$ has a sufficiently simple analytical expression or/and if the ship is symmetrical. Due to the fundamental interest in computing second order effect, it is worthwhile to explore first these simple cases.

IV. THE SPECTRAL FUNCTIONS FOR SEPARABLE SHAPE FUNCTIONS AND SYMMETRICAL SHIPS

1. Separable Shape Functions

If the function $f(x,z)$ is sufficiently regular it may be expressed under quite general conditions as a separable sum

$$f(x,z) = \sum_{k=1}^{k=n} h_k(x)m_k(z) \qquad [4.1]$$

where trigonometric or polynomial functions have been generally used for representing h_k and m_k. Considering, for the sake of simplicity, only one term of [4.1], written generically as $h(x)m(z)$, leads to the following expressions of the functions related to the spectral functions of Chapters II and III:

(i) The first order function I_1 [2.18] becomes

$$I_1(\alpha,\beta) = \int_{-\ell}^{\ell} d\bar{x}\,\frac{dh}{d\bar{x}}\,e^{i\alpha\bar{x}} \int_{-t(x)}^{0} m(\bar{z})e^{\gamma\bar{z}}\,dz \qquad [4.2]$$

(ii) The functions b^s [3.11], b_j^b [3.23, 3.27, $j = I, \dots V$) and b^ℓ [3.32], which are the only ones depending on the ship shape, may be written as follows

$$\begin{pmatrix} b^s \\ b_j^b \\ b^\ell \end{pmatrix} = \frac{i}{4\pi^2}\int_{-\ell}^{\ell} d\bar{x}\,\frac{dh}{d\bar{x}}\,e^{i(\alpha-\nu)\bar{x}} \int_{-\ell}^{\ell} \frac{dh}{d\bar{\bar{x}}}\,e^{i\nu\bar{\bar{x}}} \int_{-t(\bar{x})}^{0} d\bar{z} \int_{-t(\bar{\bar{x}})}^{0} d\bar{\bar{z}} \begin{pmatrix} T^s(\alpha,\beta,\nu,\tau,\bar{z},\bar{\bar{z}}) \\ T_j^b(\alpha,\beta,\nu,\tau,\bar{z},\bar{\bar{z}}) \\ T^\ell(\alpha,\beta,\nu,\tau,\bar{z},\bar{\bar{z}}) \end{pmatrix} \qquad [4.3]$$

where

$$T^s = m(\bar{z})\ e^{\Gamma\bar{z}}\ m(\bar{\bar{z}})\ e^{\rho\bar{\bar{z}}} \qquad [4.4]$$

$$T^b_I = m(\bar{z})\ e^{\gamma\bar{z}}\ m(\bar{\bar{z}})\ e^{-\rho|\bar{z}-\bar{\bar{z}}|}\operatorname{sign}|\bar{z}-\bar{\bar{z}}| \qquad [4.5]$$

$$T^b_{II} = m(\bar{z})\ e^{\gamma\bar{z}}\ m(\bar{\bar{z}})\ e^{-\rho|\bar{z}-\bar{\bar{z}}|} \qquad [4.6]$$

$$T^b_{III} = T^b_{IV} = m(\bar{z})\ e^{\gamma\bar{z}}\ m(\bar{\bar{z}})\ e^{\rho(\bar{z}+\bar{\bar{z}})} \qquad [4.7]$$

$$T^b_V = T^{\ell} = m(\bar{z})\ m(\bar{\bar{z}})\ e^{\rho\bar{\bar{z}}}\ \delta(\bar{z}) \qquad [4.8]$$

and $\delta(\bar{z})$ is the Dirac function.

Hence, if the integrals in [4.3] can be carried out in closed analytical form, finding a point on one of the $C_s(\beta)$ or $S_s(\beta)$ curves requires two integrations solely, over ν and τ. However, due to the similar structure of the b functions, much computer time can be saved by a rational programming.

The functions q^s [3.10], q^b_j [3.23], q^{ℓ} [3.32], T^s [4.4] and T^b_j [4.5 - 4.8] are slowly varying functions of ν and τ. In contrast, the integration over $\bar{x}$ and $\bar{\bar{x}}$ in [4.3] renders oscillatory functions of ν; the oscillations become rapid as $\ell = \ell' g/U'^2$ becomes small. Accurate numerical integration over ν of the integrands of the spectral functions imposes a dense grid in the latter case, but considerable computer time may be saved if the integrals over τ of the slowly varying functions are evaluated by interpolation from a sparser grid.

Finally, if the contour of S is rectangular, i.e., $|x| \le \ell$, $0 > z > -t$ and t = constant, the first two integrals and the last two integrals in [4.3] separate and the function

$$L(\alpha,\nu) = \frac{1}{4\pi^2}\left(\int_{-\ell}^{\ell} d\bar{x}\,\frac{dh}{d\bar{x}}\,e^{i(\alpha-\nu)\bar{x}}\right)\left(\int_{-\ell}^{\ell} d\bar{\bar{x}}\,\frac{dh}{d\bar{\bar{x}}}\,e^{i\nu\bar{\bar{x}}}\right) \quad [4.9]$$

can be computed once for all second order terms.

2. Symmetrical Ships: $f(x,z) = f(-x,z)$

In this case the function I_1 [4.2] becomes

$$I_1(\alpha,\beta) = i\iint_S \frac{\partial f}{\partial \bar{x}}\, e^{\gamma\bar{z}} \sin \alpha\bar{x}\, d\bar{x} \quad [4.10]$$

and, as well known, the cosine spectrum function $C_1(\beta)$ vanishes, while according to [2.28]

$$S_1(\beta) = -2\iint_S \frac{\partial f}{\partial \bar{x}}\, e^{\gamma\bar{z}} \sin \alpha_p\bar{x}\, d\bar{x}\, d\bar{z} \quad [4.11]$$

As for the second order terms, the functions b^s [3.11], b_j^b [3.21, 3.27] and b^ℓ [3.32] are purely imaginary. As a result

the cosine and sine spectrum functions C_s^s and S_s^s [3.18], become equal to the first and second integrals of [3.18], respectively, and the same is true for C_s^b, S_s^b [3.29] and C_s^ℓ, S_s^ℓ [3.34]. It turns out, therefore, that computing the cosine spectrum functions requires one integration less than those necessary for the sine functions.

V. APPLICATION TO A SYMMETRICAL SHIP WITH PARABOLIC WATERLINE AND RECTANGULAR FRAMES

1. General Formulae

We consider now the particular case

$$
\begin{aligned}
&f(x,z) = h(x)\, m(z) \\
&h(x) = 1-(x^2/\ell^2)\ ; \quad m(z) = 1 \qquad (|x| < 1,\ 0 < z < -t)
\end{aligned}
\tag{5.1}
$$

with t = constant. This example has been selected because it is the simplest conceivable smooth shape and because detailed measurements have been carried out for models of such shape.

For the first order free waves spectrum [2.28, 4.2] we obtain immediately the well known expression

$$
\begin{aligned}
&S_1(\beta) = -\frac{8}{\ell}\,\frac{1-e^{-\alpha_p^2 t}}{\alpha_p^3}\left(\cos \alpha_p\ell - \frac{\sin \alpha_p\ell}{\alpha_p\ell}\right) \\
&C_1(\beta) = 0
\end{aligned}
\tag{5.2}
$$

where α_p is given by [2.22] and ζ_i^w by [2.27].

The spectral functions C_s^s and S_s^s [3.18] depend in part on the auxiliary functions q^s [3.10] and q_p^s [3.16] which are independent of the shape. The function b^s [3.11] becomes in the present case

$$b^s(\alpha,\beta,\nu,\tau) = iL(\alpha,\nu)\; M^s(\alpha,\beta,\nu,\tau) \qquad [5.3]$$

where $L(\alpha,\nu)$ is defined by [4.9] and M^s results from [4.3]. Hence, by using [5.1], we obtain in the present case

$$\begin{aligned} L(\alpha,\nu) = -\frac{2}{\pi^2 \ell^2}\Bigg\{ & \frac{1}{\nu(\alpha-\nu)}\left[\cos \alpha\ell + \cos(\alpha-2\nu)\ell\right] - \\ & -\frac{1}{\nu(\alpha-\nu)^2\ell}\left[\sin \alpha\ell + \sin(\alpha-2\nu)\ell\right] - \\ & -\frac{1}{\nu^2(\alpha-\nu)\ell}\left[\sin \alpha\ell - \sin(\alpha-2\nu)\ell\right] - \\ & -\frac{1}{\nu^2(\alpha-\nu)^2\ell^2}\left[\cos \alpha\ell - \cos(\alpha-2\nu)\ell\right]\Bigg\} \end{aligned} \qquad [5.4]$$

while M^s is given by

$$M^s(\alpha,\beta,\nu,\tau) = \int_{-t}^{0} d\bar{z} \int_{-t}^{0} d\bar{\bar{z}}\; T^s(\alpha,\beta,\nu,\tau,\bar{z},\bar{\bar{z}}) = \frac{(1-e^{-\rho t})(1-e^{-\Gamma t})}{\rho\Gamma} \qquad [5.5]$$

and $_\beta\Gamma$ are defined by [3.12].

The cosine spectral function [3.18] is, therefore, given by

$$C_s^S(\beta) = 2\pi \int_{-\infty}^{\infty} d\tau \; [L(\alpha_p,\nu_p)\; M^S(\alpha_p,\beta,\nu_p,\tau)\; q^S(\alpha_p,\beta,\nu_p,\tau) +$$

$$+ L(-\alpha_p,\nu_p)\; M^S(-\alpha_p,\beta,\nu_p,\tau)\; q_p^S(-\alpha_p,\beta,\nu_p,\tau)] \qquad [5.6]$$

whereas the sine spectral function [3.18] has the expression

$$S_s^S(\beta) = -2 \int_{-\infty}^{\infty} d\tau \int_0^{\infty} d\nu \; [L(\alpha_p,\nu)\; M^S(\alpha_p,\beta,\nu,\tau)\; q^S(\alpha_p,\beta,\nu,\tau) -$$

$$- L(-\alpha_p,\nu)\; M^S(-\alpha_p,\beta,\nu,\tau)\; q^S(-\alpha_p,\beta,\nu,\tau)] \qquad [5.7]$$

all the functions appearing in [5.6] and [5.7] being defined previously.

Similar equations are obtained for the body and waterline integral second order spectral functions. The only specific computations are required by [4.3]

$$M_j^b(\alpha,\beta,\nu,\tau) = \int_{-t}^{0} d\bar{z} \int_{-t}^{0} d\bar{\bar{z}}\ T_j^b(\alpha,\beta,\nu,\tau,\bar{z},\bar{\bar{z}}) \qquad (j=I,\ldots,V) \qquad [5.8]$$

which leads with the aid of [4.4 - 4.8] and [5.1] to

$$M_I^b = \frac{1}{\rho}\left\{-\frac{1}{\gamma-\rho}\left(e^{-\rho t}-e^{-\gamma t}\right) + \frac{1}{\gamma+\rho}\left[1-e^{-(\gamma+\rho)t}\right]\right\} \qquad [5.9]$$

$$M_{II}^b = \frac{1}{\rho}\left\{\frac{2}{\gamma}\left(1-e^{-\gamma t}\right) - \frac{1}{\gamma+\rho}\left[1-e^{-(\gamma+\rho)t}\right] - \frac{1}{\gamma-\rho}\left(e^{-\rho t}-e^{-\gamma t}\right)\right\} \qquad [5.10]$$

$$M_{III}^b = M_{IV}^b = \frac{1}{\rho(\gamma+\rho)}\left(1-e^{-\rho t}\right)\left[1-e^{-(\gamma+\rho)t}\right] \qquad [5.11]$$

$$M_V^b = M^{\ell} = \frac{1}{\rho}\left(1-e^{-\rho t}\right) \qquad [5.12]$$

Hence, with $b_j^b = iL(\alpha,\nu)M_j^b(\alpha,\beta,\nu,\tau)$ we obtain the following final expressions for the spectral functions [3.29] and [3.34]

$$C_s^b(\beta) = 4\pi\int_0^{\infty} d\tau \left\{ L(\alpha_p,\nu_p)\left[\sum_{j=IV}^{j=V} q_{j,p}^b(\alpha_p,\beta,\nu_p,\tau)M_j^b(\alpha_p,\beta,\nu_p,\tau)\right] + \right.$$

$$\left. + L(-\alpha_p,\nu_p)\left[\sum_{j=IV}^{j=V} q_{j,p}^b(-\alpha_p,\beta,\nu_p,\tau)M_j^b(-\alpha_p,\beta,\nu_p,\tau)\right]\right\} \qquad [5.13]$$

$$S_s^b(\beta) = -4\int_0^\infty d\tau\int_0^\infty d\nu \left\{ L(\alpha_p,\nu)\left[\sum_{j=I}^{j=V} M_j^b(\alpha_p,\beta,\nu,\tau)q_j^b(\alpha_p,\beta,\nu,\tau)\right] - L(-\alpha_p,\nu)\left[\sum_{j=I}^{j=V} M_j^b(-\alpha_p,\beta,\nu,\tau)q_j^b(-\alpha_p,\beta,\nu,\tau)\right]\right\} \quad [5.14]$$

$$C_s^{\ell}(\beta) = 4\pi\int_0^\infty d\tau\ [L(\alpha_p,\nu_p) + L(-\alpha_p,\nu_p)]M^{\ell}(\nu_p,\tau)q_p^{\ell}(\nu_p) \quad [5.15]$$

$$S_s^{\ell}(\beta) = -4\int_0^\infty d\tau\int_0^\infty d\nu\ [L(\alpha_p,\nu) - L(-\alpha_p,\nu)]M^{\ell}(\nu,\tau)q^{\ell}(\nu,\tau) \quad [5.16]$$

2. Discussion of the Integration Procedures

The second order spectral functions have to be computed numerically. The C_s functions [5.6], [5.13] and [5.15] are based on one integration over τ while the S_s [5.7], [5.14] and [5.16] imply a double integration. The major contribution, in an asymptotic sense, to the wave drag [2.32] results from the sine functions. Herewith are a few observations related to the integration procedures in the ν,τ plane.

(i) It is easy to ascertain that the behavior at infinity of the integrands in [5.6 - 5.16] ensures integrability. The integrands in [5.6], [5.13] and [5.15] are $O(\tau^{-2})$ for $\tau \to \infty$,

while the integrands in [5.7], [5.14] and [5.16] are $O(\rho^{-3})$ for $\rho = \sqrt{\nu^2+\tau^2} \to \infty$. It is worthwhile to point out that if a slender body type additional expansion is carried out, i.e., an expansion of $e^{-\rho t}$, $e^{-\Gamma t}$, $e^{-\gamma t}$ in a t power series in [5.5] and [5.9 - 5.12], the integrals of the spectrum functions diverge because of the limited validity of such an expansion for large ρ.

(ii) The singularity of $L(\alpha,\nu)$ [5.4] at $\nu = \alpha$ is removable. In fact, it is easy to ascertain that for $\nu-\alpha \to 0$ $L(\alpha,\nu) = O(\nu-\alpha)$, which makes the integrand finite even for q_j^b [3.23]. The singularity of [5.9] and [5.10] at $\gamma-\rho = 0$ is also removable.

(iii) The integral in the ν,τ plane rendering S_s^s [5.7] involve some principal value computations. As a matter of convenience it was found that it is preferable to carry out first integration over τ and afterwards over ν. The reason is that the oscillatory function $L(\alpha,\nu)$ may be taken before the τ integral, while the remaining functions vary slowly with τ.

The singularities of the denominator of q^s [3.10] are distributed along a few curves in the ν,τ plane, which are represented in Fig. 2.

Integrals along lines ν = constant in [5.7] may cross, therefore, up to four times the curves of Fig. 2 and at each such point an evaluation of a Cauchy principal value is needed. At the beginning we have used a procedure suggested by Landweber

(Kobus, 1967, Appendix 1) in order to compute the principal values contributions. This procedure stipulates selecting the singularity as a node in an equally spaced grid and replacing the integrand by the derivative of its regular part at this point. The application of the procedure has been found to be tedious in the present case for a few reasons: at each value of ν rezoning of the grid is necessary, the use of efficient numerical integration based on orthogonal polynomials is precluded, the vertices of the curves in Fig. 2 require special treatment and numerical computation of the derivative is time consuming.

The procedure used instead, which obviously is not the only possible one, was as follows: in an integral of the type

$$I = \int_{-\infty}^{\infty} \frac{F(\nu,\tau)}{(\nu^2-\rho)\,[(\alpha-\nu)^2-\Gamma]}\,d\tau \qquad [5.17]$$

as encountered in [5.7], the singular part has been extracted and I has been replaced by I', where

$$I' = \int_{-\infty}^{\infty} \left\{ \frac{F(\nu,\tau)}{(\nu^2-\rho)[(\alpha-\nu)^2-\Gamma]} - \frac{F'(\nu)}{\tau^2-\tau'^2} - \frac{F''(\nu)}{\tau^2-\tau''^2} - \frac{F'''(\nu)}{\tau^2-\tau'''^2} \right\} \qquad [5.18]$$

$$F'(\nu) = -\frac{2\nu^2 \ F(\nu,\tau')}{(\alpha-\nu)^2-\Gamma(\alpha,\beta,\nu,\tau')} \qquad [5.19]$$

$$F''(\nu) = -\frac{2\tau'' \ (\alpha-\nu)^2 \ F(\nu,\tau'')}{(\tau''-\beta) \ [\nu^2-\rho(\nu,\tau'')]} \qquad [5.20]$$

$$F'''(\nu) = \frac{2\tau''' \ (\alpha-\nu)^2 \ F(\nu,\tau''')}{(\tau'''-\beta) \ [\nu^2-\rho(\nu,\tau''')]} \qquad [5.21]$$

$$\tau' = \nu \sqrt{\nu^2-1} \qquad [5.22]$$

$$\tau'' = \beta + (\nu-\alpha) \sqrt{(\nu-\alpha)^2-1} \qquad [5.23]$$

$$\tau''' = \beta - (\nu-\alpha) \sqrt{(\nu-\alpha)^2-1} \qquad [5.24]$$

The integrand in [5.18] is regular at the singularities of Fig. 2, including the vertices, and any numerical procedure may be used in order to compute I' provided that a too close neighborhood of the singularity is avoided.

Similar formulae are valid for the part of S_s^s [5.7] involving $q^s(-\alpha_p,\beta,\nu,\tau)$ with α_p replaced by $-\alpha_p$.

(iv) The integrals rendering S_s^b [5.14] and S_s^{ℓ} [5.16] have the only singularities along $\tau = \tau'$ and the former procedure may be used in order to make the integral regular, only the $F'(\nu)$ [5.19] type of function being needed.

VI. PRELIMINARY NUMERICAL RESULTS

1. Numerical Procedure

We present a few preliminary results of numerical computations of second order spectrum functions. These first results are presented on a tentative basis; they have to be furtherly checked in terms of accuracy of numerical procedures and to be extended to a larger range of Froude numbers. The computer program has to be improved towards saving time.

The computer program has been written in order to determine the values of S_2^s [5.8], S_2^b [5.14] and S_2^l [5.16]. The computation of the C_2 functions, which is much easier, will be carried out in the future.

As stated previously, numerical integration has been carried out first along lines ν = constant in the ν,τ plane. A 48 points Legendre procedure has been applied repetitively to intervals of $\Delta\tau = 0.25$. The integration has been stopped if the increment of the integral on such an interval was lesser than 10^{-4} of the sum on previous intervals or after 25 intervals if the criterion was not met. In the latter case integration continued over 25 enlarged new intervals until the convergence criterion was satisfied. When the integration was stopped, the rest of the integral on the remaining τ interval up to $\tau = \infty$ was evaluated by a 24 points Laguerre procedure. The change of $\Delta\tau$ from 0.25 to 0.5 showed very little change in the values of the integrals.

The integration over ν has been carried out with a Simpson procedure, the ν interval being $\pi/80\ell$, selected such that enough points should be contained in a "wavelength" of the trigonometric functions which are present in $L(\alpha-\nu)$ [5.4]. The integration has been stopped when the absolute value of the increment was smaller than 10^{-3} of the previous sum.

Summarizing, the program admits $\ell = \ell' g/U'^2$ and $t = t' g/U'^2$ as input parameters and computes S_2^s, S_2^b and S_2^ℓ for any given value of β.

2. <u>Numerical Results for Given Draft and Froude Numbers</u>

The first runs have been carried out for $t'/\ell' = 0.3$ and for $\ell = \ell' g/U'^2 = 5$ ($F_n = 0.316$) and $\ell = 12.5$ ($F_n = 0.200$). The values of the sine spectrum functions S_1, S_2^s, S_2^b and S_2^ℓ have been computed for $\beta = 0, 0.5, 1, 1.5, 2, 3, 4, 5, 6$. In Figs. 3 and 4 we have represented the functions

$$\bar{S}_1 = \frac{\ell S_1 \alpha_p^2}{2\alpha_p^2 - 1}, \quad \bar{S}_2^{s,b,\ell} = \frac{\ell^2 \alpha_p^2 S_2^{s,b,\ell}}{2\alpha_p^2 - 1} \qquad [6.1]$$

which permit to rewrite the equation of the free waves profile [2.27] in the more convenient form

$$\zeta^W = \frac{1}{\pi} \int_{-\infty}^{\infty} [\bar{\epsilon}\, \bar{S}_1(\beta) + \bar{\epsilon}^2 \bar{S}_2(\beta)] \sin \alpha_p x \cos \beta y \, d\beta \qquad [6.2]$$

where $\bar{\epsilon} = b'/\ell'$ is the usual slenderness parameter.

Although conclusions may be drawn only on a provisory basis, it is worthwhile to discuss a few qualitative features of the curves of Figs. 3 and 4:

(i) For $F_n = 0.316$ (Fig. 3a) the free surface correction, the body correction and the waterline integral correction are of the same order of magnitude. For the smaller Froude number $F_n = 0.2$ (Fig. 4a) the free-surface effect seems to tend to dominate the other two contributions.

(ii) For $F_n = 0.316$ (Fig. 3b) $\bar{S}_2 = \bar{S}_2^s + \bar{S}_2^b + \bar{S}_2^\ell$ is roughly equal and of opposite sign to $\bar{S}_1$. Hence, the second order term diminishes the amplitude of the first order free waves and of the corresponding wave resistance. For $F_n = 0.200$ (Fig. 4b) $\bar{S}_2$ is again of opposite sign to $\bar{S}_1$, but the amplitude at the first peak of the $\bar{S}_2$ curve is roughly four times larger then that of $\bar{S}_1$.

(iii) The first order solution is valid only if $\bar{\epsilon} << 1$, let say $\bar{\epsilon} < 0.1$ for $F_n = 0.316$ and $\bar{\epsilon} < 0.025$ for $F_n = 0.2$, i.e., as well known not only $\bar{\epsilon}$ but $\epsilon = \bar{\epsilon}\ell$ has to be small as second order effects become more important at low Froude numbers.

VII. SUMMARY AND CONCLUSIONS

A systematic procedure of computing the second order spectral functions of the free waves generated by a thin ship has been derived. The procedure has been applied to the case of

a ship with parabolical waterline and rectangular frames. Preliminary numerical results indicate that the free surface, body and waterline integral second order corrections are of the same order of magnitude and neglection of part of them in order to derive inconsistent, but simpler, solutions does not seem to be justified. This conclusion has to be strengthened by extending the computations to various shapes and a large range of Froude numbers.

The accuracy of the numerical program is checked at present. Later, cosine and sine spectral functions, as well the associated wave resistance, will be computed for various Froude numbers, such that drawing more definite conclusions will become possible. These future results will be reported elsewhere.

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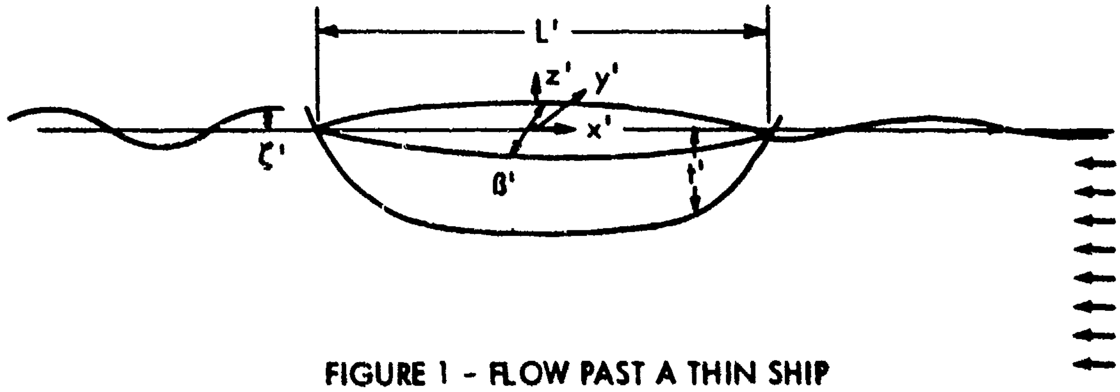


FIGURE 1 - FLOW PAST A THIN SHIP

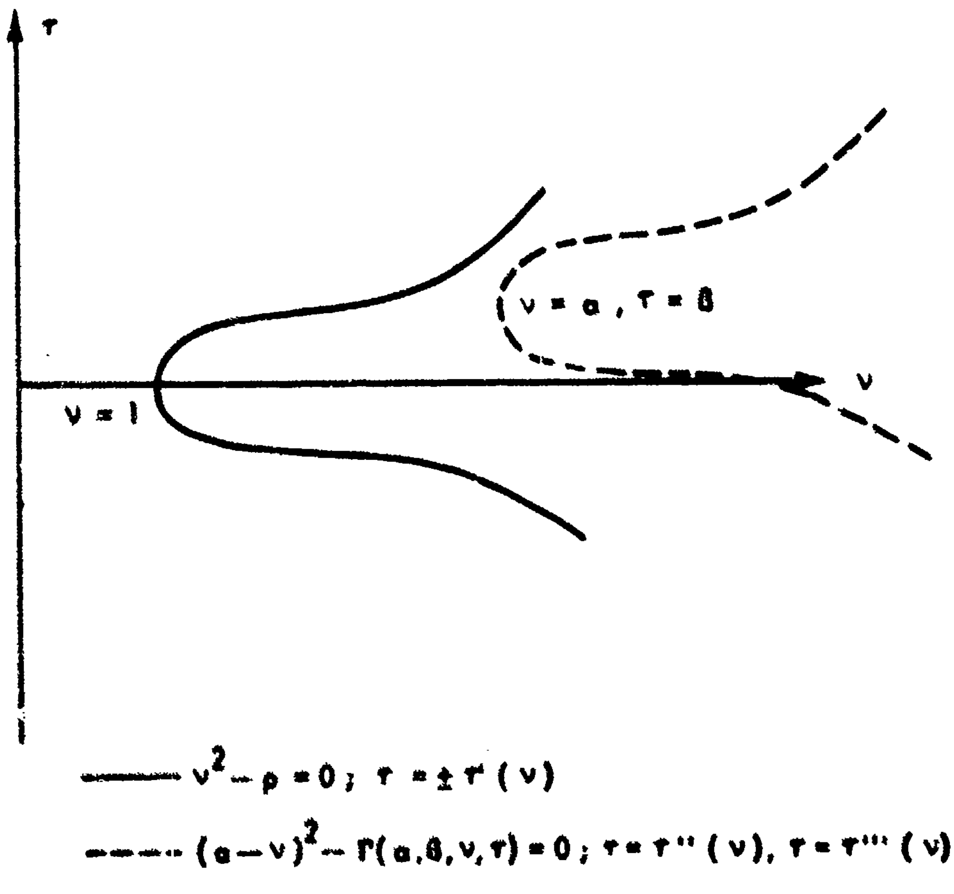
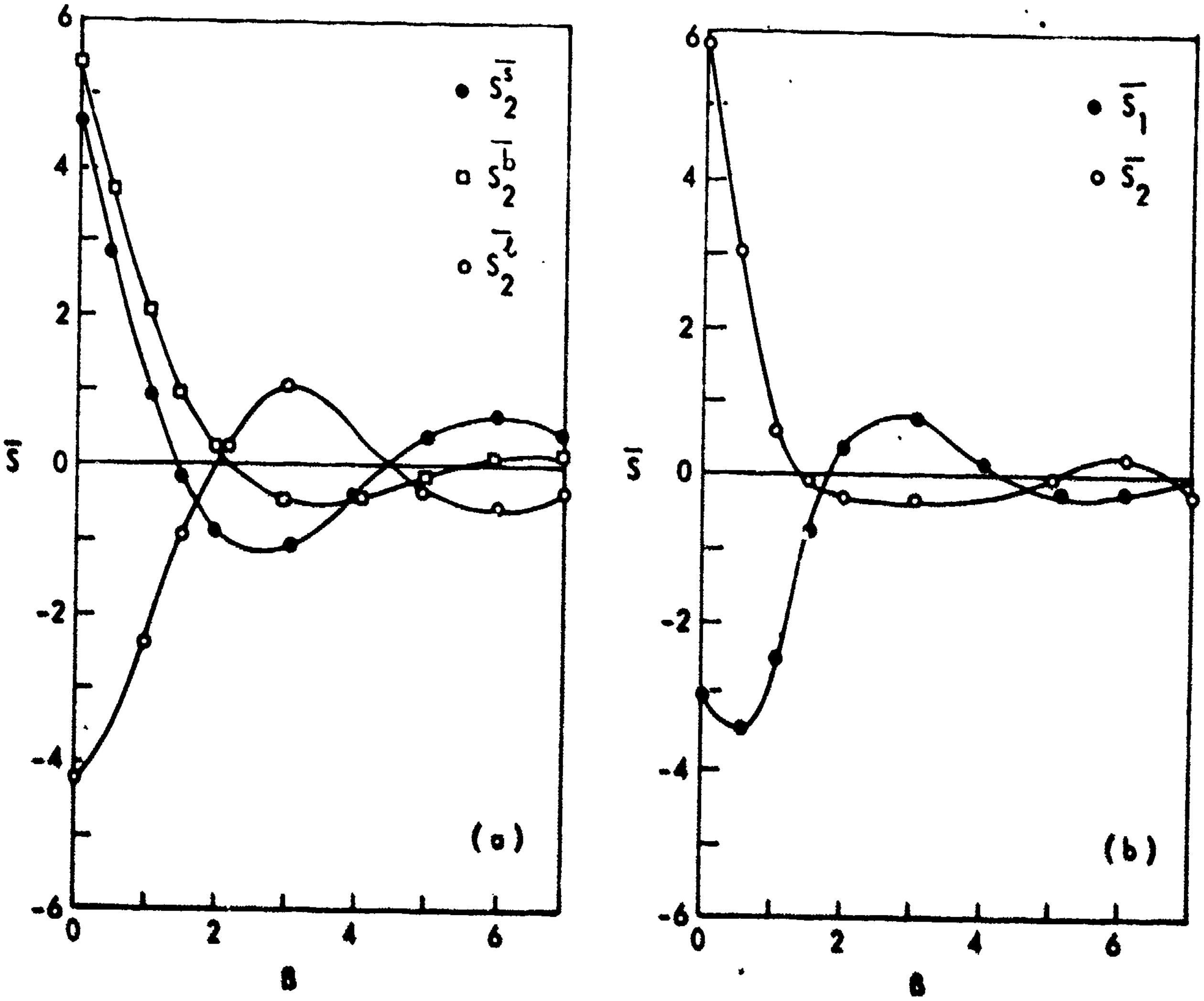


FIGURE 2 - LOCATION OF THE POLES OF $q^3(\alpha,\beta,\nu,\tau)$ [3.10] IN THE ν,τ PLANE



$$S^w = \frac{1}{\pi} \int_{-\infty}^{\infty} \left[\bar{\epsilon}\, \bar{S}_1(\beta) + \bar{\epsilon}^2 \bar{S}_2(\beta) \right] \text{SIN}\, \alpha \rho x \cos \beta y \, d\beta$$

$\bar{\epsilon} = b'/\ell'$ $\qquad$ $\bar{S}_2 = \bar{S}_2^s + \bar{S}_2^b + \bar{S}_2^\ell$

FIGURE 3 - THE SINE SPECTRUM FUNCTIONS OF THE FREE WAVES FOR $F_n = 0.316$ ($\ell = 5$)

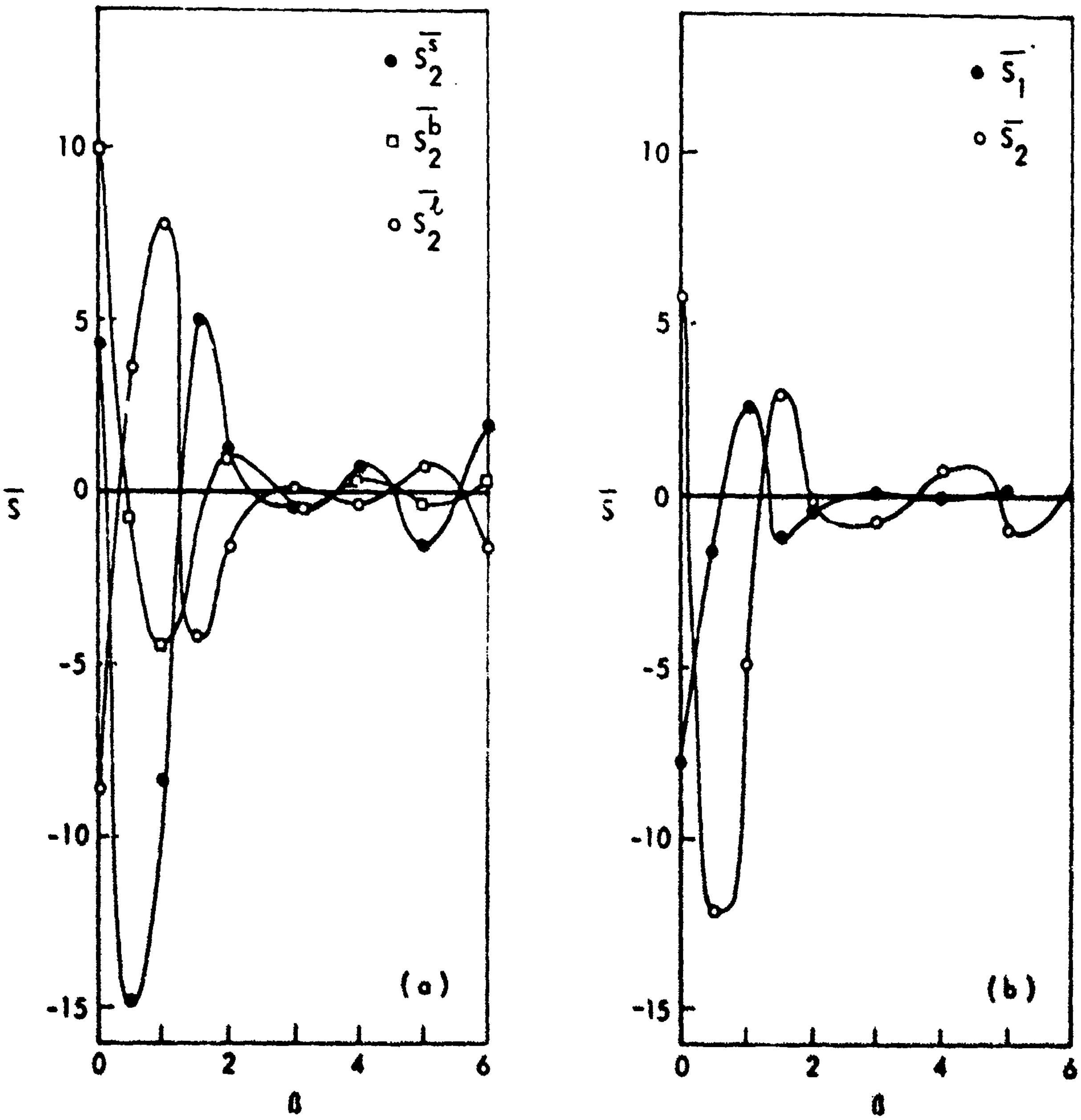


FIGURE 4 - THE SINE SPECTRUM FUNCTIONS OF THE FREE WAVES FOR $F_n = 0.200$ ($l = 12.5$)